

university of behavioural and sociology groningen social sciences

# **Statistical Analysis of Complete Social** Networks

**Co-evolution of Networks & Behaviour** 

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## Co-evolution models for networks and behaviour

- A. Interdependence of networks and behaviour
- B. Extension of the stochastic actor-based modelling framework to "behaviour" dimensions
- C. The case of homogeneity bias / network autocorrelation
- D. An example: Co-evolution of music taste, alcohol & friendship
- E. Notes on the modelling of peer influence



# A: Interdependence of networks and behavior

As could be seen already, social network dynamics can depend on actors' individual characteristics.

Some examples:

- homophily: interaction with similar others can be more rewarding than interaction with dissimilar others
- heterophily / exchange: selection of partners such that they complement own abilities and resources
- popularity: some properties make actors more attractive as network partners than other actors
- activity: some properties make actors send more network ties than other actors do



# Vice versa, also actors' characteristics can depend on the social network

Changeable individual characteristics can be affected by others in the network: behaviour proper, but also opinions, attitudes, intentions, etc. – we use the word <u>behaviour</u> for all of these! *Some examples:* 

- contagion / assimilation: innovations spreading in a professional community; adolescents adopting friends' attitudes; investment bankers copying behaviour of successful competitors
- differentiation: division of tasks in a work team
- effects of isolation: lack of connections in a network may lead to behaviour that well-connected actors do not exhibit



# There often is a "natural pairing" of effects in both directions

**Example:** Suppose "success attracts sponsors". This will lead to a positive association between success and indegree in any cross-sectional data collection. The same cross-sectional association, however, can also be explained by "sponsors make you successful".

More generally, any *cross-sectional association* between network features and individual characteristics could come about by at least two *competing mechanisms*:

- 1. The network leads to behavioural alignment.
- 2. Actors' behaviour leads to network alignment.

Aim: construction of a model that allows a teasing apart.



# **B.** Extension of the network modelling framework

- Stochastic process in the (extended!) space of all possible network-behaviour configurations



 $r^n$  states, where r is the range of the <u>ordinal</u> behaviour variable  $z^{n(n-1)}$  states in the case of a (binary) directed network variable x

- Again, the first observation is not modelled but conditioned upon as the process' starting value.
- Discrete change is modelled as occurring in continuous time, but now there are two types of change.



# Actor based approach now in two domains

- Network actors drive the process: individual decisions.
  - > <u>two</u> domains of decisions:
    - decisions about **network** neighbours,
    - decisions about own behaviour.
  - > <u>per decision domain</u> two model parts:
    - When can actor **i** make a decision? (rate functions  $\lambda^{net}$ ,  $\lambda^{beh}$ )
    - Which decision does actor  $\mathbf{i}$  make? (objective functions  $f^{net}$ ,  $f^{beh}$ )

By again sampling waiting times and identifying the shortest one, it becomes clear *who* makes *which type* of change.







## Schematic overview of model components

	<b>Timing of decisions</b>	<b>Decision rules</b>
Network evolution	Network rate function $\lambda^{net}$	Network objective function <b>f</b> <sup>net</sup>
Behavioural evolution	Behaviour rate function $\lambda^{beh}$	Behaviour objective function <b>f</b> <sup>beh</sup>

- > By simultaneously operating both processes on the same state space (conditionally independent, given the current state), *feedback processes* are instantiated.
- > Processes of network evolution and of behavioural evolution therefore are *controlled for each other's occurrence*!



## Micro steps that are modelled explicitly

Let  $(\mathbf{x},\mathbf{z})(\mathbf{t})$  be the state of the co-evolution process at time point  $\mathbf{t}$  (where  $\mathbf{x}$  stands for the network part and  $\mathbf{z}$  for the behaviour vector).

Micro steps are defined as "smallest possible changes":

#### network micro steps

 $(\mathbf{x},\mathbf{z})(\mathbf{t}_1)$  and  $(\mathbf{x},\mathbf{z})(\mathbf{t}_2)$  differ in one tie variable  $\mathbf{x}_{ij}$  only.

behaviour micro steps

 $(x,z)(t_1)$  and  $(x,z)(t_2)$  differ by one in one behavioural score variable  $z_i$  only.



# Model for behavioural change

## **Choice options:**

(1) increase, (2) decrease, or (3) keep current score on the ordinal behavioural variable, provided the range is not left



#### **Choice probabilities:**

Analogous to network part: multinomial logit model based on evaluations of options according to behavioural objective function.

### Explanatory model for behaviour change:

By inclusion of effect statistics in the objective function.



	effect	network statistic	effective transitions in network*	verbal description
	1. tendency	z	$\circ \leftrightarrow \bullet$	main behavioral tendency
	<ol> <li>indegree</li> <li>× behavior</li> </ol>	$\mathbf{z}_i \sum_j \mathbf{x}_{ji}$	•⊷ ↔ •⊷•	effect of own popularity on behavior
Also here,	<ol> <li>outdegree</li> <li>× behavior</li> </ol>	$\mathbf{z}_i \sum_j \mathbf{x}_{ij}$	⊶• ↔ •⊷∘	effect of own activity on behavior
many effects	<ol> <li>dense triads</li> <li>× behavior</li> </ol>	$\mathbf{z_i} \sum\nolimits_{jh} \texttt{group}(ijh)$		effect of belonging to cohesive subgroups on behavior
are possible	5. peripheral × behavior	$\mathbf{z}_i \sum\nolimits_{jhk} \texttt{peripheral}(i; jhk)$		effect of being peripheral to cohesive subgroups on behavior
to include in	6. isolation × behavior	$\boldsymbol{z}_i \text{ isolate}(i)$	ې → بَ	effect of being isolated in the network on behavior
	7. similarity	$\sum_{j} \mathbf{x}_{ij} \sin_{ij}$		assimilation to friends (contagion / influence) $% \left( \left( {{{\mathbf{x}}_{i}}} \right) \right)$
the objective	<ol> <li>similarity</li> <li>x reciprocity</li> </ol>	$\sum\nolimits_{j} x_{ij} x_{ji}  \text{sim}_{ij}$		assimilation to reciprocating friends
function	9. similarity × pop. alter	$\sum\nolimits_{j} {{{\mathbf{x}}_{ij}}} \sin _{ij} \sum\nolimits_{k} {{{\mathbf{x}}_{kj}}}$	ooo ←→ ooo ooo ←→ ooo	assimilation to popular friends
	10. similarity × dense triads	$\sum\nolimits_{jh} group(ijh)(\text{sim}_{ij} + \text{sim}_{ih})$	$\left(\begin{array}{c} 25 \\ 25 \end{array}\right) \mapsto \left(\begin{array}{c} 25 \\ 25 \end{array}\right)$	assimilation to the majority behavior in a cohesive subgroup
	11. similarity × peripheral	$ \sum_{jhk} (peripheral(i;jhk) \\ \times (sim_{ij} + sim_{ik} + sim_{ik})) $		assimilation to those cohesive subgroups one unilaterally attaches to

TABLE 3 SELECTION OF POSSIBLE EFFECTS FOR MODELING BEHAVIORAL EVOLUTION

\* In the effective transitions illustrations, it is assumed that the behavioral dependent variable is dichotomous and centered at zero; the color coding is Q = low score (negative),

=high score (positive), O = arbitrary score. Actor i is the actor who changes color z, in the transition indicated by the double arrows. Illustrations are not exhaustive.





d by yFiles



# Estimation of co-evolution models

- The estimating equations algorithm needs to be modified slightly because the default equations for 'competing process explanations' are identical and would imply an unsolvable collinear system of equations.
- Solution: work with *cross-lagged statistics* in the estimating equations!
  - Network change in response to prior behaviour,
  - behaviour change in response to prior network.



# Estimating equations

When **X**, **Z** are model-based simulated data and **x**, **z** the empirical data, the following statistics are used:

- > For parameters in the network objective function:  $S(X,Z) = \sum_{k} \sum_{i} s_{ih}^{net} (X(t_{k+1}), z(t_{k}))$
- > For parameters in the behaviour objective function:  $S(X,Z) = \sum \sum e^{beh} (x(t_{-}), Z(t_{--}))$

 $\mathbf{S}(\mathbf{X},\mathbf{Z}) = \sum_{k} \sum_{i} \mathbf{s}_{ih}^{beh}(\mathbf{x}(\mathbf{t}_{k}),\mathbf{Z}(\mathbf{t}_{k+1}))$ 

The estimating equations are E(S(X,Z)) = S(x,z); everything else remains as in the case of the simple network evolution model.



# **C. Explaining homogeneity bias**

In networks connected actors are often behaviourally more similar than non-connected actors. Technically, this has been termed *homogeneity bias* or *network autocorrelation*.



One measure (implemented in SIENA) is the *network similarity statistic*  $\sum_{j} x_{ij} \sin_{ij}$ , where  $\sin_{ij}$  is a standardised measure of similarity of two actors based on their distance on a variable z,  $\sin_{ij} = 1 - (|z_i - z_j| / range_z)$ .  $\sin_{ij} = 1$  means scores of i and j are identical;  $\sin_{ij} = 0$  means

they are maximally apart (one maximal, the other minimal).



## Competing explanatory stories

Actors base their social relations on similarity of individual features.



Actors adjust their individual features to the features of their social environment.





# Modelling selection and influence

By including the network similarity statistic  $\sum_{j} x_{ij} \sin_{ij}$ 

...in the <u>network objective function</u>, homophilous selection is modelled,

...in the *behaviour objective function*, assimilation / social influence is modelled.

It can be of crucial importance to be able to control one effect for the occurrence of the other – e.g., in the design of social interventions to reduce smoking at school.



# An advantage of / reason for continuous time modelling

Suppose in a given data set, transition Network change other Behavior change other (a) on the right has than homophily than assimilation been observed from one observation moment to the next. May one diagnose Assimilation Homophily this observation as possible possible occurrence of assimilation?

The continuous time approach allows to control for other explanations such as (b)-(c)-(d); discrete time models cannot do this!



## **D: Example co-evolution analysis\***

A set of illustrative research questions:

- 1. To what degree is music taste acquired via friendship ties?
- 2. Does music taste (co-)determine the selection of friends?
- 3. What is the role played by alcohol consumption in both friendship evolution and the dynamics of music taste?
- Data: Medical Research Council's *Teenage Friends & Lifestyle Study* (Bush, Michell & West, 1997) three waves, 129 pupils (13-15 year old) at one Glasgow-based school; pupils named up to 6 friends
- \* see Steglich, Snijders & West, *Methodology* 2: 48-56 (2006)



43. Which of the following types of music do you like listening to? Tick one or more boxes. Rock Indie Chart music  $\Box$ Jazz Classical Reggae Dance 60's/70's *Heavy Metal*  $\Box$ House  $\square$ Techno  $\Box$ Grunge Folk/Tradit.  $\Box$ Rap Rave  $\Box$ Hip Hop Other (what?).....

Before applying SIENA: data reduction to informative dimensions...

# Principal components analysis (confirmed by Mokken scaling) yields three music listening dimensions...





#### Alcohol question: five point scale





#### Average dynamics of the four behavioural variables...





#### ...and global dynamics of friendship (dyad counts)





# Analysis of the music taste data

### Network objective function:

- intercept:outdegree
- covariate-determined:
   gender homophily
   gender ego
   gender alter
- **Rate functions** were kept as simple as possible (periodwise constant).

- network-endogenous:
   reciprocity distance-2
- behaviour-determined:
   beh. homophily
   beh. ego
   beh. alter

"behaviour" stands shorthand for the three music taste dimensions and alcohol consumption.



#### **Behaviour objective function(s):**

- intercept:

### tendency

– network-determined:

## assimilation to neighbours

– covariate-determined:

### gender main effect

– behaviour-determined:

behaviour main effect

The following slides show the original estimation results (2006, Steglich, Snijders & West).



		parameter	s.e.	t-score	<b>Results:</b> network evolution
outdegre	е	-1.89	0.29	-6.51	
reciproci	ty	2.34	0.12	20.08	Low overall density in
distance	-2	-1.09	0.07	-14.89	these networks
gender	sim	0.80	0.12	6.72	these networks.
	alter	-0.21	0.12	-1.73	Paginrogation is important for
	ego	0.24	0.11	2.17	friendship
techno	sim	0.08	0.33	0.26	Inclusinp.
	alter	0.07	0.05	1.30	
	ego	-0.10	0.05	-1.93	transitive closure
rock	sim	0.11	0.41	0.26	transitive closure.
	alter	0.19	0.07	2.75	
	ego	-0.07	0.08	-0.92	
classical	sim	1.44	0.69	2.07	
	alter	0.15	0.17	0.91	
	ego	0.40	0.17	2.42	
alcohol	sim	0.83	0.27	3.08	
	alter	-0.03	0.04	-0.75	
	eqo	-0.03	0.03	-0.85	



		parameter	s.e.	t-score	Results: network evolution
outdegre	е	-1.89	0.29	-6.51	-
reciprocit	ty	2.34	0.12	20.08	There is gender homophily:
distance-	2	-1.09	0.07	-14.89	alter
gender	sim	0.80	0.12	6.72	boy girl
	alter	-0.21	0.12	-1.73	boy 0.38 -0.62
	ego	0.24	0.11	2.17	ego
techno	sim	0.08	0.33	0.26	- gill -0.16 0.41
	alter	0.07	0.05	1.30	table gives gender-related
	ego	-0.10	0.05	-1.93	contributions to the objective function
rock	sim	0.11	0.41	0.26	-
	alter	0.19	0.07	2.75	There is alcohol homophily:
	ego	-0.07	0.08	-0.92	alter
classical	sim	1.44	0.69	2.07	low high
	alter	0.15	0.17	0.91	low 0.36 -0.59
	ego	0.40	0.17	2.42	high -0.59 0.13
alcohol	sim	0.83	0.27	3.08	
	alter	-0.03	0.04	-0.75	table shows contributions to the objective function for highest / lowest
	ego	-0.03	0.03	-0.85	possible scores



		parameter	s.e.	t-score	Results: network evolution
outdegre	е	-1.89	0.29	-6.51	
reciproci	ty	2.34	0.12	20.08	
distance-	2	-1.09	0.07	-14.89	Techno style listeners are
gender	sim	0.80	0.12	6.72	marginally less active in
	alter	-0.21	0.12	-1.73	sending friendship
	ego	0.24	0.11	2.17	nominations.
techno	sim	0.08	0.33	0.26	Pock style listeners are
	alter	0.07	0.05	1.30	more popular as
	ego	-0.10	0.05	-1.93	friends.
rock	sim	0.11	0.41	0.26	
	alter	0.19	0.07	2.75	Classical style listeners
	ego	-0.07	0.08	-0.92	select each other as
classical	sim	1.44	0.69	2.07	friends!
	alter	0.15	0.17	0.91	
	ego	0.40	0.17	2.42•	Classical style listeners are
alcohol	sim	0.83	0.27	3.08	friendship nominations
	alter	-0.03	0.04	-0.75	mendship nominations.
	ego	-0.03	0.03	-0.85	



#### **Results: behavioural evolution**

	alcohol		techno		rock		classical	
	par.	s.e.	par.	s.e.	par.	s.e.	par.	s.e.
intercept	-0.30	0.37	0.01	0.25	0.59	0.25	0.67	1.30
assimilation	0.94	0.27	0.45	0.18	0.63	0.28	0.42	1.17
gender	-0.06	0.19	0.25	0.12	0.01	0.19	1.57	0.83
techno	0.23	0.16			-0.25	0.09	-0.46	0.40
rock	0.16	0.16	-0.34	0.10			0.64	0.39
classical	-0.59	0.32	-0.13	0.23	-0.34	0.30		
alcohol			0.07	0.10	-0.11	0.07	-1.03	0.34

• Assimilation to friends occurs: -on the alcohol dimension,

- on the techno dimension,

– on the rock dimension.



#### **Results: behavioural evolution**

	alcohol		tecl	techno		rock		classical	
	par.	s.e.	par.	s.e.	par.	s.e.	par.	s.e.	
intercept	-0.30	0.37	0.01	0.25	0.59	0.25	0.67	1.30	
assimilation	0.94	0.27	0.45	0.18	0.63	0.28	0.42	1.17	
gender	-0.06	0.19	0.25	0.12	0.01	0.19	1.57	0.83	
techno	0.23	0.16			-0.25	0.09	-0.46	0.40	
rock	0.16	0.16	-0.34	0.10			0.64	0.39	
classical	-0.59	0.32	-0.13	0.23	-0.34	0.30			
alcohol			0.07	0.10	-0.11	0.07	-1.03	0.34	

- There is evidence for mutual exclusiveness of:
  - listening to techno and listening to rock,
  - listening to classical and drinking alcohol.
- The classical listeners tend to be girls.



## Time for a hands-on exercise!





# E: More on peer influence modelling

- Peer influence doesn't necessarily mean "connected people becoming / staying more similar over time"
  - For strongly skewed variables, peer influence may even coincide with connected people becoming less similar.

<u>Example:</u> When entering secondary school, students initially are all non-delinquent, i.e., perfectly similar. Any subsequent movement implies a reduction of similarity.

 In such cases, the *similarity based* measures can be wrong specifications of peer influence!
 Correlational measures may be the better choice here; see Knecht et al. (Social Development, 2010) & following slides.



# Minor delinquency & friendship

### A <u>negative</u> effect of 'getting similar' was estimated!

- > Huh?
- <u>Do students want to</u> <u>differ from their</u> <u>friends</u>, on the delinquency dimension?
- > Take closer look!





#### A. Very skewed distribution.





#### **B.** Increase over time



If the dynamic process starts with perfect similarity ("nobody delinquent") it can only get <u>less</u> similar from there on...





The positive alignment of alter averages and ego scores suggests there <u>might be</u> influence, after all... just not of the sort captured in the distance-based similarity measure!



#### Use a correlational measure for social influence instead!





## To remember:

- > "Influence" is <u>not</u> unequivocally tied to one specific operationalisation!
- > It is <u>not always</u> about "similarity" sometimes
   "alignment" / "association" is the better way to phrase
   it and sometimes it is a "connectedness" issue.
- > <u>Always</u> take a close look at your data set to find out what makes sense in your context.
- > In the stochastic actor-based framework, goodness of fit tests (score type) facilitate the technical part of decision making – but doesn't substitute thought!



## Next issue: Consider distributional shape!

 The simple 'intercept' or 'tendency' (now 'linear shape') parameter used by Steglich, Snijders & West (2006) is not a good baseline model for behaviour variables:



 > It can <u>only</u> express monotonous, not too extremely skewed baseline distributions as the result of behaviour change in the long run.

## *But... empirical distributions often are <u>unimodal</u> or <u>U-shaped</u>!*



#### Why is this a problem?

> If a distributional shape persists over time, this stability will be captured by parameter estimates.

<u>Example:</u> If a behaviour variable is empirically over- or underdispersed with respect to its best-fitting 'linear shape' model, the residual dispersion can bias peer influence estimates.

An illustration is the paper by Baerveldt et al., 2008.

- Best is to work with empirically meaningful baseline distributions – including U-shapes and unimodality.
   U-shape or strong skewness are cases of overdispersion; unimodality is a case of underdispersion w.r.t. the linear shape model.
- > So... enhance 'baseline capabilities' of the behaviour model!



#### The 'quadratic shape' parameter

 The addition of a 'quadratic shape' parameter allows the modelling of also unimodal, U-shaped, and strongly skewed baseline distributions as long-run result of behaviour change:



> Note, however, that there still can be other, weird empirical distributional shapes! *Always check, recode if too weird!* 



#### Interpretation of 'quadratic shape' estimates

- Besides the rather technical dispersion interpretation, the 'quadratic shape' parameter can be interpreted as follows:
  - <u>positive sign:</u> *"The higher the behaviour already is, the higher the tendency to increase it even more."* Change dynamics self-accelerating towards extremes. Behaviour is potentially 'addictive'. Polarisation of the group on this behaviour dimension is likely.
  - <u>negative sign:</u> *"The higher the behaviour already is, the lower the tendency to still increase it further."* Change dynamics self-correcting towards the mean. Behaviour is potentially governed by norms of moderation that hold in the whole group. Consensus formation on this behaviour is likely.



# **Some results obtained with this model** (Steglich, Snijders & Pearson, 2010)

Shape (see next slide) does not necessarily dominate influence.

	ALCOHOL			SMOKING			
	estimate	st.error	p-value	estimate	st.error	p-value	
shape: linear	0.41	( 0.14 )	0.004	-2.61	( 0.42 )	< 0.001	
shape: quadratic	0.01	( 0.11 )	0.926	2.62	( 0.31 )	< 0.001	
average similarity	6.70	( 2.18 )	0.002	2.63	( 1.06 )	0.014	

- > Alcohol: "real influence" (network partners attract)
- > Smoking: both influence and general polarisation trend.



# Distribution of the corresponding variables





# What about the distributional shapes of the four behaviour variables in Steglich, Snijders & West (2006)?





#### Robustness check of results reported by Steglich, Snijders & West upon addition of 'quadratic shape' effect to the model

#### quadratic shape parameters:

- > weakly negative (p=0.08) for *alcohol consumption* (unimodal)
- > positive (p=0.01) for *classical / elite* (strongly skewed)
- > n.s. (p>0.6) for *rock* and *techno / chart*

#### change in peer influence results:

- > result for rock drops to n.s. (p=0.16)
- > result for *techno / chart* drops to weak effect (p=0.08)

#### change in homophily-based selection:

result for classical / elite drops to weak effect (p=0.08)
 Overall "slightly less spectacular results", it seems.



#### Interpretation of robustness check results

- > The overall drop in significance of almost all effects can be a result of adding four more parameters to an already large model, which implies a reduction of statistical power.
- > The strongest drop in significance occurs for the 'assimilation rock' effect: Controlling for the whole cohort's behavioural tendencies, it is not possible to tell anymore whether friends adjusted their rock listening habits to those of their friends.
- > Besides these comments, the new results seem in line with the earlier reported ones.



## To remember:

- > "Network influence" can be diagnosed while actually there is no network level influence operating!
- Control for distributional shape of the behaviour variable
   otherwise the model might pick it up with its network
   influence parameter, i.e., sell you shape as influence.
- Results show both patterns can be identified in appropriate data sets.

### But how to <u>interpret</u> this quadratic parameter? Can we attach a meaning (e.g., social norm)?





The *quadratic shape effect* expresses the squared distance from the global average of the behaviour variable (if centered). Modify it a bit...





**Example** (Torlò, Lomi, Snijders & Steglich, 2010)

Advice seeking & performance among 75 MBA students in Rome



#### **Performance distribution**









# In this data set, network influence is indistinguishable from group level influence!

Results for a specification with network influence only:

PERFORMANCE	estimate	st.error	p-value
tendency	-0.236	0.113	0.037
distance to advisors	-0.599	0.179	<0.001

Results for a specification controlling for group influence:

PERFORMANCE	estimate	st.error	p-value
tendency	0.546	0.679	0.421
distance to advisors	-0.389	0.250	0.119
distance to social norm	-0.413	0.358	0.249



# Wrapping up

Main messages:

- Assessment of peer influence effects requires control for peer selection effects
- As far as possible, also context effects need to be taken into account.
  - Validity of results obtained by stochastic actor-based modelling is conditional on having all relevant variables inside the model specification!
- > The model assumption of *decomposability into smallest possible steps* is crucial for separating peer influence and peer selection effects.



## Further aspects not covered in this presentation

The issue of <u>comparing the strength</u> of influence and selection .

 Requires joint metric for comparison, e.g. *network autocorrelation coefficients* (Moran, Geary); see

> Steglich, Snijders & Pearson, 2010. Dynamic Networks and Behavior: Separating Selection from Influence. *Sociological Methodology* 40: 329-393.



SMOKING MODELS

ALCOHOL MODELS



## An autocorrelation fit measure

- > To what degree are performance of advice giver and advice recipient associated?
- > Indicator Moran's autocorrelation:

 $I = \frac{n \sum_{ij} x_{ij} (z_i - \overline{z}) (z_j - \overline{z})}{\left(\sum_{ij} x_{ij}\right) \left(\sum_i (z_i - \overline{z})^2\right)}$ 

 Compared are (partially) nested models including these components: T rend (rewiring, perf. drift, etc.)
 C ontrol (sex, experience, etc.)
 S election (homophily, etc.)
 I nfluence (assimilation, etc.)

