

INTRODUCTION TO  
DYNAMIC SOCIAL NETWORK ANALYSIS

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## Research on network dynamics

- \* Mathematics: (purely) random graphs (Erdős - Renyi)
- \* Computer science, theoretical physics:  
study of effects of simple, but not completely random, rules  
in infinitely growing networks (e.g., www)
- \* Sociology: interaction between individuals  
(friendship, collaboration, sex)  
but also other social actors – organizations, countries.
- \* Economics: choices by individuals with interdependent payoffs  
game theory – strategic behavior;  
equilibrium, stable networks.
- \* Biology: interacting species, interacting proteins.

## *Statistical (inferential) modeling of networks*

- ⇒ allows to *test* theories about network development, about mutually interdependent development of networks and behavior;
- ⇒ allows generalization from empirical data to conclusions about populations.

### Characteristics of statistical paradigm

(may be evident, but can be useful to make explicit):

#### *care about uncertainty in conclusions*

- ⇒ due to random deviations (“*s.e., tests*”)
- ⇒ due to incorrect auxiliary assumptions (“*robustness, misspecification*”)

This is important especially for non-experimental research, because there it is so difficult to ‘control’ for other effects.

Example of network-related research question:

*How strongly do smoking habits of high-school pupils depend on SES (controlling for A, B, C, ...)?*

If influence between friends plays a role,  
then assuming independence between schoolmates  
is a misspecification ;

e.g., a few popular pupils in a school class  
may have a relatively large influence.

(Similar methodological issues occur in multilevel analysis,  
where a given number of 'cases' may give less information  
in clustered data than in independent data.)

If friendship groups tend to be homogeneous in SES, then the SES effect will be associated with the friendship effect.

⇒ friendship is a *nuisance factor* which should be controlled for.

However: when going beyond factor models to explanatory models,

⇒ friendship is an *interesting factor* which must be modeled jointly with the smoking behavior, because of the two-way influence smoking ⇔ friendship.

Misspecification of the model for friendship may bias the conclusions about social influence, as well as the conclusions about effects of variables, such as SES, that are associated with friendship (e.g. by homophily).

## Methodological research program

- ⇒ how to model network dynamics
- ⇒ how to model joint networks & behavior dynamics
- ⇒ how robust are conclusions to misspecification

work in progress — leads to new perspectives on old questions, and to new questions, new wishes for data collection;

requires painstaking modeling,  
collaboration methodologists – social scientists.

Misspecification is an issue that has not yet been addressed sufficiently.

## Example

Gerhard van de Bunt, Rafael Wittek, and Maurits de Klepper,  
The Evolution of Intra-Organizational Trust Networks;  
The Case of a German Paper Factory:  
An Empirical Test of Six Trust Mechanisms.  
*International Sociology* 20 (2005), 339-369.

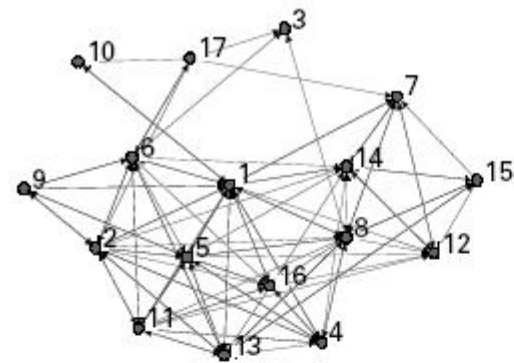
Dynamics of networks (without behavior)

Research question:

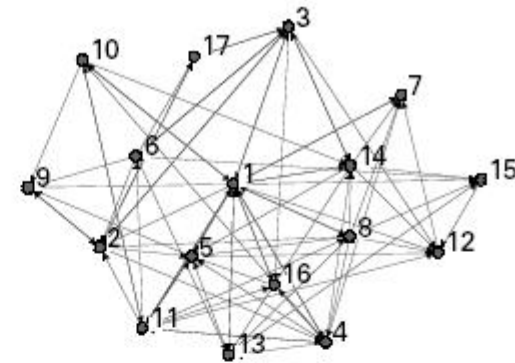
*what determines formation of trust in a work setting?*

Case study of paper factory.

One directed network: trust; 17 persons, 4 waves.

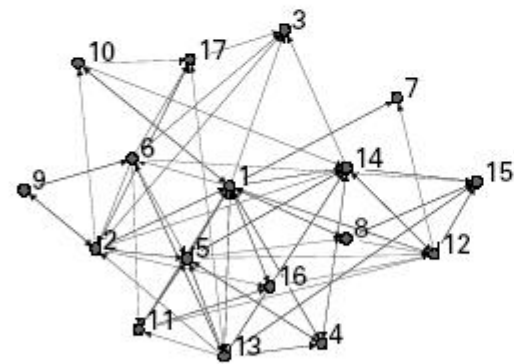


December 1995 (time =  $t_1$ )

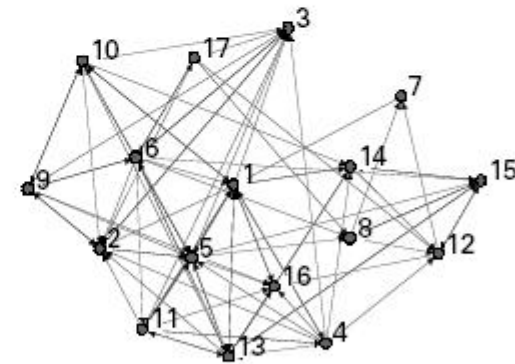


July 1996 (time =  $t_2$ )

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December 1996 (time =  $t_3$ )



July 1997 (time =  $t_4$ )

**Figure 1** *The Trust Network at Four Points in Time*

A trust relationship from ego to alter is present if ego perceives the relationship to either be strong or very strong.



## Actor-oriented approach to network dynamics

The repeated network observations are regarded as discrete observations of a process developing in continuous time, where actors make unobserved changes between observations, constituting each others' changing environment.

The changing network acts as a dynamic constraint on each actors' behavior.

Each actor “controls” his/her outgoing relations;  
in the case of joint dynamics of networks and behavior,  
the actor “controls” also his/her own behavior.

The actors try to attain a rewarding position in the network.  
The appreciation by actor  $i$  of his/her position in the network  $x$   
is expressed by the *objective function*  $f_i(x)$ .  
The objective, or aim, of actor  $i$  is  
to achieve a high value of the objective function  $f_i(x)$ .

Actions propelled also by a *random component*,  
expressing *unexplained change* (‘residual term’).

There are separate objective functions for network change and for behavior change.

These objective functions are the focus of the model construction.

At any given moment, with a given current network structure, the actors act independently, without coordination.

They also act one-at-a-time.

No strategy;

objective functions reflect

short-time goals, opportunities, constraints,

At stochastically determined moments, actors get the opportunity to make one change in either their outgoing ties; or in their behavior.

The difference between two consecutively observed networks is achieved by a sequence of such small (unobserved) changes: *micro-steps*.

Next to the objective function(s), another component of the model construction are the rates at which the actors make changes in their outgoing ties, or in their behavior.

These rates can be constant or changing: *rate functions*.

These subsequent changes generate endogenous dynamic context which implies a dependence between the actors over time; e.g., through reciprocation or transitive closure one tie may lead to another one.

Thus, there is a strong dependence between what the actors do, but it is generated not by joint decision making or anticipation of each other's behavior (like e.g. in game theory), but entirely by the time order of their decisions, and the fact that the actors constitute each others' changing environment.

## Back to the example: trust in the paper factory

*What are essential elements of how people determine and change their social ties?*

1. number of ties 'outdegree'
2. reciprocity
3. transitivity:  
*I trust you; you trust her; then I will also trust her*
4. balance:  
*if we trust each other, we trust the same third parties*
5. characteristics of individuals: actor variables, covariates

Transitivity and balance are very similar:  
different implementations of the same basic idea.

Data:

network ties are measured as binary (0-1) variables.

Start with a baseline model:

Objective function is a weighted sum  
of number of ties (outdegree) and  
number of reciprocated ties.

The weights are the estimated parameters.

Rates of changes are constant between observations.

**Table 4** *Baseline Trust Model: Estimated Parameters of the Transition from a Neutral Trust Relationship to a Trust Relationship*

Parameters	Baseline model
$\lambda_{23}$	5.43 (0.91)**
$\lambda_{34}$	4.86 (0.86)**
$\delta$	-0.86 (0.17)**
Reciprocity	1.15 (0.22)**

Standard errors within parentheses.

\*  $p < .05$ ; \*\*  $p < .01$ .

Estimations are based on 2000 simulation runs.

*Objective function:*

$$(-0.86 \times \text{outdegree}) + (1.15 \times \# \text{ reciprocated ties})$$

Constant rates of change:

on average 5.43, 4.86 change opportunities between observations.



Next, three theoretically argued models and a final expressive model.

Actor covariates can be included in the objective function in a variety of ways. Three basic ways:

1. value for ego = sender of tie;
2. value for alter = receiver of tie;
3. similarity between ego and alter.

**Table 5** *Homophily Model, Balance Model, Gossip Model and the Final Expressive Model: Estimated Parameters of the Transition from a Neutral Trust Relationship to a Trust Relationship*

Parameters	Homophily model <sup>a</sup>	Balance model	Gossip model	Final expressive model
$\lambda_{23}$	5.95 (1.02)**	5.36 (0.85)**	5.83 (0.97)**	5.85 (0.96)**
$\lambda_{34}$	5.35 (0.97)**	4.76 (0.84)**	5.23 (0.92)**	5.25 (0.96)**
$\delta$	-0.85 (0.12)**	-0.76 (0.17)**	-0.87 (0.13)**	-0.79 (0.15)**
Reciprocity	1.07 (0.20)**	1.13 (0.22)**	1.17 (0.21)**	1.07 (0.21)**
Tenure (in years)	-0.63 (0.35)*			
Age (in years)	0.90 (0.40)*			
Level of education	-0.03 (0.18)			
Hierarchical level	0.05 (0.36)			
Trust in peers	0.12 (0.45)			
Trust in management	-0.14 (0.34)			
Structural balance		0.98 (0.49)*		0.79 (0.48)*
Gossip sim.			0.05 (0.39)	
Gossip $\times$ pop. alter			1.82 (0.86)*	1.69 (0.82)*

**Table 6** *The Instrumental Models: Estimated Parameters of the Transition from at Most a Neutral Relationship to at Least a Strong Trust Relationship*

Parameters	Signalling model	Sharing group model	Structural holes model	Final instrumental model
$\lambda_{23}$	5.28 (0.94)**	5.90 (1.03)**	5.33 (0.85)**	4.77 (0.78)**
$\lambda_{34}$	4.64 (0.91)**	5.61 (1.11)**	5.17 (0.95)**	4.68 (0.91)**
$\delta$	-1.22 (0.56)**	-0.20 (0.33)	-1.00 (0.19)**	-0.91 (0.18)**
Reciprocity	1.05 (0.22)**	1.03 (0.22)**	1.14 (0.25)**	1.08 (0.25)**
Sup. → sub.	1.23 (0.52)**			
Task dependency		0.92 (0.37)**		
Dyadic constraint			1.09 (0.22)**	1.17 (0.23)**
Efficiency ego			3.44 (1.39)**	3.84 (1.77)**

Standard errors within parentheses.

\* $p < .05$ ; \*\* $p < .01$ .

Estimations are based on 2000 simulation runs. All models include the baseline model.

**Table 7** *The Final Trust Model:<sup>a</sup> Estimated Parameters of the Transition from at Most a Neutral Relationship to at Least a Strong Trust Relationship*

Parameters	Final trust model	Parameters	Elaborated final trust model
$\lambda_{23}$	5.68 (0.92)	$\lambda_{23}$	5.94 (1.02)**
$\lambda_{34}$	5.56 (1.02)	$\lambda_{34}$	5.95 (1.06)**
$\delta$	-0.99 (0.14)	$\delta$	-1.02 (0.15)**
Reciprocity	1.10 (0.22)	Reciprocity	1.23 (0.25)**
Dyadic constraint	1.07 (0.18)**	Dyadic constraint	0.65 (0.27)**
		Breaking constraining tie	-0.97 (0.58)*
Efficiency ego	3.13 (1.30)**	Efficiency similarity	-0.14 (0.61)
		Efficiency alter	-1.06 (0.92)
		Efficiency ego	3.60 (1.27)**

<sup>a</sup> Except for the structural holes effects, all effects are non-significant.

Standard errors within parentheses.

\* $p < .05$ ; \*\* $p < .01$ .

Estimations are based on 2000 simulation runs. All models include the baseline model.

Table 7 ('elaborated final model') contains an additional element: asymmetry between tie creation and tie termination.

Due to investment in relations,  
or experience with relations  
that was not obtained before they existed,  
breaking off an existing tie can be different from  
the opposite of creating a new tie.

This is expressed by the *endowment effect* of ties  
(something lost when the tie is broken,  
that was not gained immediately when it was created.)  
(Other terminology: gratification effect,  
gratification inherent in the act of creating / breaking a tie.)

## Note on parameter interpretation

The parameters are weights in objective (& endowment) functions, not direct tendencies to change.

The objectives of the social actors (individuals) will *lead to change* if the currently existing situation is far from what is optimal for the actors, given the remainder of the network; and they will *maintain* a structure in dynamic equilibrium if the existing situation is close to optimal.

As the objective function is optimized myopically, without strategic foresight, this is a model of boundedly rational goal pursuit rather than a utility model as in microeconomics. This is even more strongly so in the models for networks and behavior.

## Networks and Behavior

*Simultaneous endogenous dynamics of networks and behavior:* e.g.,

- \* individual humans & friendship relations:  
attitudes, behavior (lifestyle, health, etc.), behavioral norms
- \* individual humans & cooperation relations:  
work performance, contributions to collective goods
- \* companies / organisations & alliances, cooperation:  
performance, organisational success
- \* sexually transmitted diseases and sexual contacts;  
but this will require somewhat different models.

## *Two-way influence between networks and behavior*

Relational embeddedness is important for well-being, behavior, opportunities, etc.; cf. studies of *social capital*.

Also, actors are influenced in their behavior and attitudes by other actors to whom they are tied (e.g., N. Friedkin, *A Structural Theory of Social Influence*, C.U.P., 1998).



In addition, many types of tie (friendship, cooperation, liking, etc.) are influenced positively by similarity on relevant attributes: *homophily* (e.g., McPherson, Smith-Lovin, & Cook, *Ann. Rev. Soc.*, 2001.)

More generally, actors choose relation partners on the basis of their behavior and other characteristics (similarity, opportunities for future rewards, etc.).

## Terminology:

relation = network = pattern of ties in group of actors;  
behavior = any individual-bound changeable attribute  
(including attitudes, performance, etc.).

*Influence*, network effects on behavior;  
e.g., assimilation / contagion.

*Selection*, behavior effects on relations; e.g., homophily.

Relations and behaviors are endogenous variables  
that develop in a simultaneous dynamics.

### *Elements of theories :*

- ⇒ Risky social behaviors (like smoking, taking alcohol or drugs) are 'contagious' among friends but also operative in friendship formation.  
Learning; social status; imitation, uncertainty reduction; interdependence in production of wellbeing.
- ⇒ How hard pupils and employees work is subject to social control.  
Interdependent payoffs; norms; exchange between friendship and contributions.
- ⇒ Firms choose partners for collaboration to achieve better results.  
Resources (complementariness); trust; reputation; information.

Thus, there is a feedback relation in the dynamics of relational networks and actor behavior / performance.

The investigation of such social feedback processes is difficult:

- \* Both the *network*  $\Rightarrow$  *behavior* and the *behavior*  $\Rightarrow$  *network* effects lead to an association between current behavior and network: “friends of smokers are smokers” (cf. work by Baumann, Kirke), “high-reputation firms don’t collaborate with low-reputation firms”. It is hard to ascertain the strengths of the causal relations in the two directions.
- \* For many phenomena quasi-continuous longitudinal observation is infeasible. Instead, it may be possible to observe networks and behaviors at a few discrete time points. (The more, the better....) *Such an observation design is the point of departure here.*

## Data:

One bounded set of actors  
(e.g. school class, group of professionals, set of firms);

several observation moments;

for each observation moment:

⇒ one network: who is tied to whom (binary)

⇒ behavior of all actors (ordered discrete);

more than one dependent behavior variable is possible.

Aim: disentangle effects *networks* ⇒ *behavior*  
from effects *behavior* ⇒ *networks*.

## Actor-oriented models:

Actors control not only their outgoing ties but also their behavior. Implicit in these models is that there is a great deal of *inertia* in the network as well as the behavior: changes are made only if the actors 'decide' accordingly.

Network change process and behavior change process run simultaneously, and influence each other being each other's changing constraints.

For network and each dependent behavior variable, there are

1. objective function
2. (endowment function)
3. rate function.

*Network dynamics* can depend on

1. endogenous network effects (reciprocity, transitivity, ... )
2. exogenous actor variables
3. dependent  $\sim$  endogenous actor variables; e.g., homophily.

*Behavior dynamics* can depend on

1. basic tendency & exogenous actor variables
2. network position of ego
3. behavior of others depending on their network connections (to ego, but perhaps also to others);  
e.g., assimilation, contagion
4. ego's values on other dependent behavior variables.

## Example:

Christian Steglich, Tom Snijders, and Patrick West (2005)  
Applying SIENA: An illustrative analysis of the co-evolution of  
adolescents' friendship networks, taste in music,  
and alcohol consumption.

To appear in *Methodology*

<http://www.ppsw.rug.nl/steglich/pdf/SteglichSnijdersWest.pdf>



## Example:

Christian Steglich, Tom Snijders, and Mike Pearson (2004).  
Dynamic Networks and Behavior:  
Separating Selection from Influence.

[http://stat.gamma.rug.nl/snijders/SSP\\_total.pdf](http://stat.gamma.rug.nl/snijders/SSP_total.pdf)

Study of smoking initiation and friendship in Scottish secondary school

(following up on earlier work by P. West, M. Pearson & others, see Pearson & Michell, *Drugs: educ., prev. and policy*, 2000.)

One school year group from a Scottish secondary school starting at age 12-13 years, was monitored over 3 years, 129 pupils present at all 3 observations, with sociometric & behavior questionnaires at three moments, at appr. 1 year intervals.

What does this data set tell us about the mutual effects between friendship and smoking?

How to explain the generally observed *network autocorrelation* between friendship and smoking status: What is evidence for selection and for influence, if both are simultaneously considered in the model?

It turned out that also considering alcohol consumption led to deeper insights into the friendship-smoking dynamics.

If sufficiently rich data is available, questions can be asked about the more precise ways in which social influence and social selection take place.

In principle, any network measures could be used.

This will allow a combination of the rich tradition in Social Network Analysis of network indices with the principles of inferential statistics.

E.g., hypotheses such as:

- \* actors imitate structurally similar others, not necessarily related others.
- \* actors imitate popular others
- \* imitation is stronger in one-sided than in reciprocal friendship ties
- \* actors imitate others who are embedded in cohesive subgroups

## Some further issues

1. Relations with more than 2 ordered values.
2. Multiple (multivariate, multiplex) relations.
3. Other influence models, e.g., contagion.
4. Assessment of model fit, misspecification questions.
5. Explained variation ( $R^2$ ) .
6. Better methods of estimation.
7. Emergent properties, micro  $\Rightarrow$  macro questions.
8. Computing time.

## Multilevel network analysis

This whole exposition has been in terms of one single group. However, for theory testing and for generalizable propositions, it is desirable to analyze empirical data on a *sample* of groups.

This requires not only data collection on multiple groups, but also the appropriate analysis methods.

A meta-analysis of groupwise network analyses was proposed by Snijders & Baerveldt (J. Math. Soc. 2003).

Stronger approaches to multilevel network analysis, based on random coefficient models like the HLM, are in preparation.