Simulation-based Statistical Inference for Social Network Dynamics

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- etc.......

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- alliances between firms
- alliances and conflicts between countries
- etc.......

These can be represented mathematically by graphs or more complicated structures.

Examples of research questions:

- ¿ Is there a tendency toward transitivity?
  (‘friends of my friends are my friends’)
- ¿ Does ethnic background have an effect on friendship, controlling for reciprocity and transitivity?
- ¿ what is the role of friendship between adolescents in smoking initiation?
- ¿ Is advice giving / receiving related to status?
- ¿ Is there a hierarchy in advice?
- ¿ Do strategic alliances follow earlier contacts between board members?

In some of such questions, networks are *independent variables*.
This has been the case in many studies
for explaining well-being (etc.);
this later led to studies of network resources,
social capital, solidarity,
in which the network is also a *dependent variable*.

Networks are dependent as well as independent variables:
intermediate structures in macro–micro–macro phenomena.

Here: focus first on networks as dependent variables,
then on mutual dependence networks and behavior
('behavior' stands here also for other individual attributes).

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Single observations of networks are snapshots,
the results of untraceable history.
Everything depends on everything else.

Therefore, explaining them has limited importance.
Longitudinal modeling offers more promise for understanding.
The future depends on the past.

Recently, important progress has been made in modelling single (‘cross-sectional’) observations in networks
– *exponential random graph models* –
this is an interesting subject, but not the topic of this lecture.

The well-known basic type of statistical modeling
of linear regression analysis and its generalizations
cannot be transplanted to network analysis,
where the focus has to be on *modeling dependencies*.

Instead, longitudinal statistical modeling of networks
relies heavily on *modest process modeling*:
use models for network dynamics that can be simulated
as models for data
– even though direct calculations are infeasible.

1. Networks as dependent variables

Repeated measurements on social networks:
at least 2 measurements (preferably more).

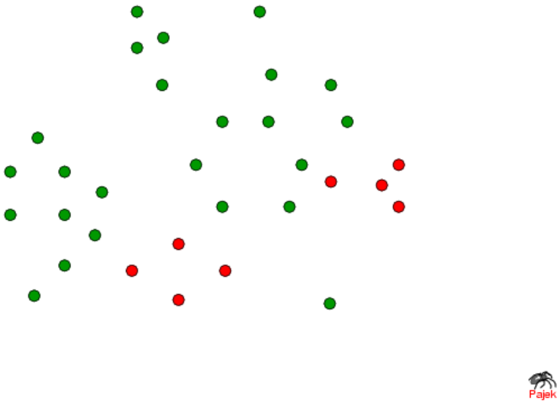
Data requirements:

The repeated measurements must be close enough together,
but the total change between first and last observation
must be large enough
in order to give information about rules of network dynamics.

Example: Studies Gerhard van de Bunt

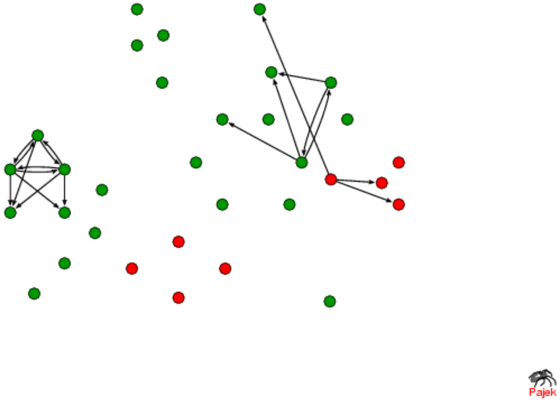
1. Study of 32 freshman university students,
   7 waves in 1 year.
   See van de Bunt, van Duijn, & Snijders,
   Computational & Mathematical Organization Theory,
   5 (1999), 167 – 192.
2. Study of hospital employees,
   2 departments (49 and 30 actors), 4 waves.

This presentation concentrates on the first data set, which can be pictured by the following graphs (arrow stands for 'best friends').



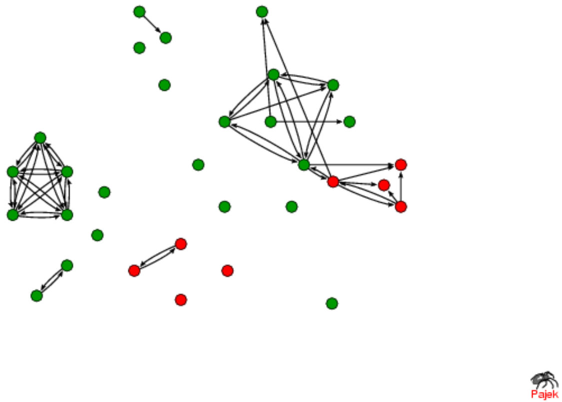
Friendship network time 1.

Average degree 0.0; missing fraction 0.0.



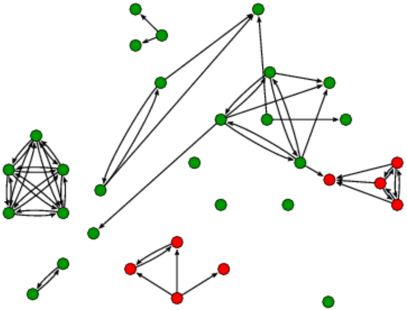
Friendship network time 2.

Average degree 0.7; missing fraction 0.06.



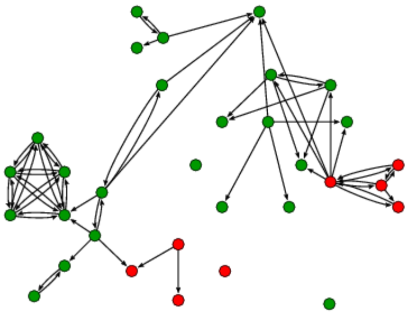
Friendship network time 3.

Average degree 1.7; missing fraction 0.09.



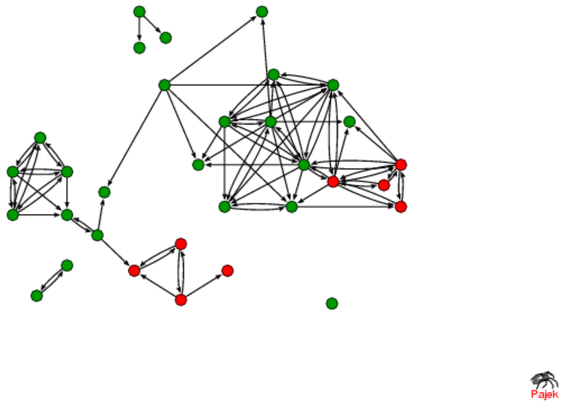
Friendship network time 4.

Average degree 2.1; missing fraction 0.16.



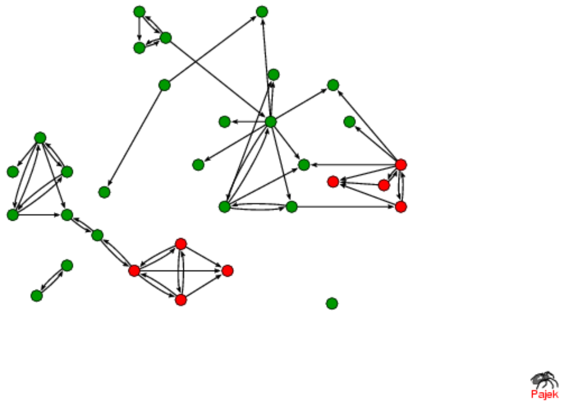
Friendship network time 5.

Average degree 2.5; missing fraction 0.19.



Friendship network time 6.

Average degree 2.9; missing fraction 0.04.



Friendship network time 7.

Average degree 2.3; missing fraction 0.22.

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Dynamics of social networks are complicated
because “network effects” are endogenous feedback effects:
e.g., reciprocity, transitivity, popularity, subgroup formation.

For statistical inference, we need models for network dynamics
that are flexible enough to represent
the complicated dependencies in such processes;
while satisfying also the usual statistical requirement
of parsimonious modelling:
complicated enough to be realistic,
not more complicated than empirically necessary and
justifiable.

For a correct interpretation of empirical observations about network dynamics collected in a panel design, it is crucial to consider a model with *latent change* going on between the observation moments.

E.g., groups may be regarded as the result of the coalescence of relational dyads helped by a process of transitivity ("friends of my friends are my friends").
Which groups form may be contingent on unimportant details; *that* groups will form is a sociological regularity.

Therefore:
use dynamic models with *continuous time parameter*:
time runs on between observation moments.

An advantage of using continuous-time models,
even if observations are made at a few discrete time points,
is that a more natural and simple representation may be found,
especially in view of the endogenous dynamics.
(cf. Coleman, 1964).

No problem with irregularly spaced data.

For *discrete data*: cf. Kalbfleisch & Lawless, JASA, 1985;
for *continuous data*:
mixed state space modelling well-known in engineering,
in economics e.g. Bergstrom (1976, 1988),
in social science Tuma & Hannan (1984), Singer (1990s).

Purpose of statistical inference:

investigate network evolution *(dependent var.)* as function of

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2. explanatory actor variables *(independent vars.)*
3. explanatory dyadic variables *(independent vars.)*

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all controlling for each other.
By controlling adequately for structural effects, it is possible to test hypothesized effects of variables on network dynamics (without such control these tests would be unreliable).

The structural effects imply that the presence of ties is highly dependent on the presence of other ties.

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3. use methods of statistical inference
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4. for panel data: employ a continuous-time model
   to represent unobserved endogenous network evolution
5. condition on the first observation and do not model it:
   no stationarity assumption.

Notation and assumptions

1. *Actors* $i = 1, \ldots, n$ (individuals in the network), pattern X of *ties* between them : one binary network X; $X_{ij} = 0$, or 1 if there is no tie, or a tie, from i to j.

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   actor-dependent covariates v, dyadic covariates w.
   These can be constant or changing over time.
3. Continuous time parameter t,
   observation moments $t_1, \ldots, t_M$.
4. The current state of the network $X(t)$
   acts as a dynamic constraint for its own process of change:
   Markov process.

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   At any single moment in time,
   only one variable $X_{ij}(t)$ may change.
7. Changes are made by the actors to optimize their situation, as it will obtain immediately after this change.
8. Random element in assessment by actors of their situation expresses aspects not modeled explicitly.

Some of these assumptions will be relaxed in future work.

(7) and (8): goal-directed behavior,
in the weak sense of myopic stochastic optimization.

Assessment of the situation is represented by
objective function, interpreted as
'that which the actors seem to strive after in the short run'.

Next to actor-driven models,
also tie-driven models are possible.

At any given moment, with a given current network structure,
the actors act independently, without coordination.
They also act one-at-a-time.

The subsequent changes ('micro-steps') generate
an endogenous dynamic context
which implies a dependence between the actors over time;
e.g., through reciprocation or transitive closure
one tie may lead to another one.

This implies strong dependence between what the actors do,
but it is completely generated by the time order:
the actors are dependent because they constitute
each other's changing environment.

Further elaboration

At randomly determined moments t,
actors i have opportunity to change a tie variable X_{ij}:
micro step.
(Actors are also permitted to leave things unchanged.)
Frequency of micro steps is determined by *rate functions*.

When a micro step is taken,
the actor optimizes an *objective function*
which is the sum of a deterministic and a random part.
The random part reflects unexplained variation.

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The distinction between rate function and objective function
separates the model for *how many* changes are made
from the model for *which* changes are made.

Specification: rate function

'how fast is change / opportunity for change ?'

If the rate of change of the network by actor *i* is λ_i ,
this means that, for a certain short time interval $(t, t + \epsilon)$,
the probability that this actor randomly gets an opportunity
to change one of his/her outgoing ties, is given by $\epsilon \, \lambda_i$.

Rate functions can depend on observation period $(t_{m-1},\ t_m)$,
actor covariates, actor behavior, and network position,
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Simple specification: rate functions are constant within periods.

Network rate functions could also depend on covariates,
degrees, etc.

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   expressing aspects of satisfaction with network
   that are obtained 'free' but are lost at a value
   (to allow asymmetry between creation and deletion of ties);
3. *random variable* with a Gumbel distribution
   leading to probabilities as in multinomial logit modeling.

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The evaluation and endowment functions express how the dynamics of the network process depends on its current state.

Evaluation function and endowment function modeled
as linear combinations of theoretically argued components
of actors' assessment of the network.
The weights in the linear combination are
the statistical parameters (cf. regression coefficients).

The focus of modeling is first on the evaluation function;
then on the rate and endowment functions.

Example: SIENA applet.

Micro-step

At random moments (frequency determined by rate function),
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to make a change in one tie variable:
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This actor tries to improve his/her objective function
and looks only to its value immediately after this micro-step
(*myopia*) .

This absence of strategy or farsightedness in the model
implies the *definition* of effects as
“what the actors try to achieve in the short run”.

Simple model specification:

- The actors all receive opportunities to change a tie at random moments, at the same rate ρ.

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- The actors all receive opportunities to change a tie at random moments, at the same rate ρ.
- Each actor tries to optimize an *evaluation function* with respect to the network configuration,

$$f_i(\beta, x)\,, \quad i = 1, ..., n, \;\; x \in \mathcal{X}\,,$$

which indicates the preference of actor i for the relational situation represented by x; objective function depends on *parameter* β.

Whenever actor i may make a change,
he changes at most one relation, say x_{ij}.

The new network is denoted by $x(i \rightsquigarrow j)$.

Actor i chooses the "best" j by maximizing

$$f_i\big(\beta, x(i \rightsquigarrow j)\big) \quad + \quad U_i(t, x, j)\,.$$

$$\Uparrow$$

random component

It is permitted to leave the network unchanged;
represented formally by $j = i$; thus, $x(i \rightsquigarrow i) = x$.

For a convenient distributional assumption,
(U has type 1 extreme value = Gumbel distribution)
given that i is allowed to make a change,
the probability that i changes the tie variable to j,
or leaving the tie variables unchanged, is

$$p_{ij}(\beta, x) = \frac{\exp\big(f(i,j)\big)}{\sum_{h=1}^{n} \exp\big(f(i,h)\big)}$$

where

$$f(i,j) = f_i\big(\beta, x(i \rightsquigarrow j)\big)$$

and p_{ii} is the probability of not changing anything.

This is the multinomial logit form of a *random utility* model.

Intensity matrix

This specification implies that X follows a *continuous-time Markov chain* with intensity matrix

$$q_{ij}(x) = \lim_{dt \downarrow 0} \frac{\mathrm{P}\{X(t+dt) = x(i \leadsto j) \mid X(t) = x\}}{dt} \qquad (i \neq j)$$

given by

$$q_{ij}(x) = \lambda_i(\alpha, \rho, x)\, p_{ij}(\beta, x)\,.$$

Computer simulation algorithm for arbitrary rate function $\lambda_i(\alpha, \rho, x)$

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$$\lambda_+(\alpha, \rho, x) = \sum_i \lambda_i(\alpha, \rho, x) \ .$$
3. Select randomly $i \in \{1, ..., n\}$ using probabilities
$$\frac{\lambda_i(\alpha, \rho, x)}{\lambda_+(\alpha, \rho, x)} \ .$$

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6. Go to step 2 (unless stopping criterion is satisfied).

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Simple specification: only evaluation function.

Evaluation function f_i reflects network effects (endogenous) and covariate effects (exogenous).

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or dyad-dependent: w_{ij} .

Convenient definition of evaluation function is a weighted sum

$$f_i(\beta, x) = \sum_{k=1}^{L} \beta_k \, s_{ik}(x) \, ,$$

where the weights β_k are statistical parameters indicating strength of effect $s_{ik}(x)$.

Choose possible network effects for actor i, e.g.:
(others to whom actor i is tied are called here i's 'friends')

1. *out-degree effect*, controlling the density,
   $s_{i1}(x) = x_{i+} = \sum_j x_{ij}$

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3. *popularity effect*, sum of in-degrees of i's friends
   $s_{i3}(x) = \sum_j x_{ij}\, x_{+j} = \sum_j x_{ij} \sum_h x_{hj}$

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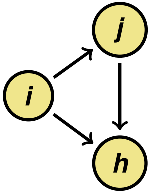
4. *activity effect*, sum of the out-degrees of i's friends
   $s_{i4}(x) = \sum_j x_{ij}\, x_{j+} = \sum_j x_{ij} \sum_h x_{jh}$

Three effects related to network closure:

5. *transitivity effect*,
number of transitive patterns
in i's ties
$(i \rightarrow j,\ j \rightarrow h,\ i \rightarrow h)$
$s_{i5}(x) = \sum_{j,h} x_{ij}\, x_{jh}\, x_{ih}$



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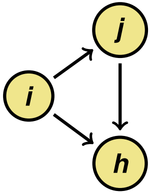

transitive triplet

6. *indirect ties effect*,
   number of actors j to whom i is tied indirectly
   (through at least one intermediary: $x_{ih} = x_{hj} = 1$)
   but not directly $x_{ij} = 0$),
   = number of geodesic distances equal to 2,
   $s_{i6}(x) = \#\{j \mid x_{ij} = 0,\ \max_h(x_{ih}\, x_{hj}) > 0\}$

7. *balance* or structural equivalence,
similarity between outgoing ties of i
with outgoing ties of his friends,

$$s_{i7}(x) = \sum_{j=1}^{n} x_{ij} \sum_{\substack{h=1 \\ h \neq i,j}}^{g} \big(1 - |x_{ih} - x_{jh}|\big) \ ,$$

[note that $\big(1 - |x_{ih} - x_{jh}|\big) = 1$ if $x_{ih} = x_{jh}$,
and 0 if $x_{ih} \neq x_{jh}$, so that

$$\sum_{\substack{h=1 \\ h \neq i,j}}^{g} \big(1 - |x_{ih} - x_{jh}|\big)$$

measures agreement between i and j .]

Differences between these three network closure effects:

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Differences between these three network closure effects:

- transitive triplets effect: i more attracted to j if there are *more* indirect ties $i \rightarrow h \rightarrow j$;
- negative indirect ties effect: i more attracted to j if there is *at least one* such indirect connection ;
- balance effect: i prefers others j who make same choices as i.

Non-formalized theories usually do not distinguish between these different closure effects.
It is possible to 'let the data speak for themselves' and see what is the best formal representation of closure effects.

Three kinds of evaluation function effect associated with actor covariate v_i :

8. *covariate-related popularity*, 'alter'
   sum of covariate over all of i 's friends
   $s_{i8}(x) = \sum_j x_{ij} v_j$;

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   $s_{i8}(x) = \sum_j x_{ij} \, v_j$;
9. *covariate-related activity*, 'ego'
   i's out-degree weighted by covariate
   $s_{i9}(x) = v_i \, x_{i+}$;

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9. *covariate-related activity*, 'ego'
   i's out-degree weighted by covariate
   $s_{i9}(x) = v_i x_{i+}$;
10. *covariate-related similarity*,
    sum of measure of covariate similarity
    between i and his friends, e.g., if V has range 0 – 1
    $s_{i10}(x) = \sum_j x_{ij} \left(1 - |v_i - v_j|\right)$

Evaluation function effect for dyadic covariate w_{ij} :

11. *covariate-related preference*,
    sum of covariate over all of i's friends,
    i.e., values of w_{ij} summed over all others to whom i is tied,
    $s_{i11}(x) = \sum_j x_{ij}\, w_{ij}$.
    If this has a positive effect, then the value of a tie $i \rightarrow j$ becomes higher when w_{ij} becomes higher.

Example

Data collected by Gerhard van de Bunt:
group of 32 university freshmen,
24 female and 8 male students.

Three observations used here (t_1, t_2, t_3) :
at 6, 9, and 12 weeks after the start of the university year.
The relation is defined as a 'friendly relation'.

Missing entries $x_{ij}(t_m)$ set to 0
and not used in calculations of statistics.

Densities increase from 0.15 at t_1 via 0.18 to 0.22 at t_3 .

Very simple model: only out-degree and reciprocity effects

	Model 1	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	3.26	(0.48)
Rate $t_2 - t_3$	2.83	(0.42)
Out-degree	-1.03	(0.19)
Reciprocity	1.76	(0.27)

rate parameters:
per actor about 3 opportunities for change between observations;

out-degree parameter negative:
on average, cost of friendship ties higher than their benefits;

reciprocity effect strong and highly significant ($t = 1.76/0.27 = 6.5$).

Evaluation function is

$$f_i(x) = \sum_j \Big(-1.03\, x_{ij} + 1.76\, x_{ij}\, x_{ji} \Big).$$

This expresses how much actor i likes the network.

Adding a reciprocated tie (i.e., for which $x_{ji} = 1$) gives

$$-1.03 + 1.76 = 0.73.$$

Adding a non-reciprocated tie (i.e., for which $x_{ji} = 0$) gives

$$-1.03,$$

i.e., this has negative benefits.

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$$f_i(x) = \sum_j \Big(-1.03\, x_{ij} \; + \; 1.76\, x_{ij}\, x_{ji} \Big) .$$

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$$-1.03 ,$$

i.e., this has negative benefits.

Gumbel distributed disturbances are added:

these have variance $\pi^2/6 = 1.645$ and s.d. 1.28.

Conclusion: reciprocated ties are valued positively,
unreciprocated ties negatively;
actors will be reluctant to form unreciprocated ties;
by ‘chance’ (the random term),
such ties will be formed nevertheless
and these are the stuff on the basis of which
reciprocation by others can start.

(Incoming unreciprocated ties, $x_{ji} = 1, x_{ij} = 0$ do not play a role
because for the objective function
only those parts of the network are relevant
that are under control of the actor,
so terms not depending on the outgoing relations of the actor
are irrelevant.)

Structural model including two network closure effects

	Model 2	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	3.87	(0.57)
Rate $t_2 - t_3$	3.12	(0.48)
Out-degree	−1.45	(0.26)
Reciprocity	1.90	(0.33)
Transitive triplets	0.22	(0.12)
Indirect ties	−0.32	(0.07)

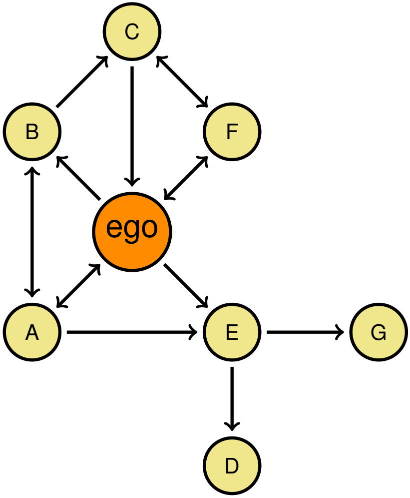
Both network closure effects are significant
(controlling for each other!),
but negative indirect ties effect is stronger.

For an interpretation, consider the simpler model without the transitive triplets effect. The estimates are:

Structural model with one network closure effect

	Model 3	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	3.82	(0.60)
Rate $t_2 - t_3$	3.17	(0.52)
Out-degree	−1.05	(0.19)
Reciprocity	2.43	(0.40)
Indirect ties	−0.55	(0.08)

Example: Personal network of ego.



for ego:

out-degree $x_{i+} = 4$

#{recipr. ties} = 2,

#{at distance 2} = 3.

The evaluation function is

$$f_i(x) = \sum_j \Big(-1.05\, x_{ij} + 2.43\, x_{ij}\, x_{ji} - 0.55\, (1 - x_{ij})\, \max_h \big(x_{ih}\, x_{hj}\big)\Big)$$

$\Big($ note: $\sum_j (1 - x_{ij})\, \max_h \big(x_{ih}\, x_{hj}\big)$ is #{indiv. at distance 2} $\Big)$

so its current value for this actor is

$$f_i(x) = -1.05 \times 4 + 2.43 \times 2 - 0.55 \times 3 = -0.99.$$

Some options for actor ‘ego’ when ego can make a ministep:

	out-degree	recipr.	distance 2	gain
current	4	2	3	
new tie to C	5	3	2	+1.93
new tie to D	5	2	2	-0.50
new tie to G	5	2	2	-0.50
drop tie to A	3	1	4	-1.93
drop tie to F	3	1	3	-1.38
drop tie to E	3	2	1	+2.15

The actor adds random influences to this (with s.d. 1.28), and chooses the change with the best total value.

Effects of sex and program, smoking similarity

	Model 3	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	4.05	(0.66)
Rate $t_2 - t_3$	3.10	(0.47)
Out-degree	−1.55	(0.24)
Reciprocity	1.83	(0.32)
Transitive triplets	0.21	(0.11)
Indirect ties	−0.30	(0.07)
Sex (M) alter	0.57	(0.25)
Sex (M) ego	−0.39	(0.27)
Sex similarity	0.35	(0.25)
Program similarity	0.38	(0.13)
Smoking similarity	0.37	(0.20)

Conclusion:

Men more popular;

No significant
sex similarity effect.

To interpret the three effects of actor covariate *gender*,
it is more instructive to consider them simultaneously.
Gender was coded originally by with 0 for F and 1 for M
but this dummy variable was centered
(the mean was subtracted)
which led to scores $z_i = -0.25$ for F and $+0.75$ for M.

The joint effect of the gender-related effects
for the tie variable x_{ij} from i to j is

$$-0.39\, z_i + 0.57\, z_j + 0.35\, I\{z_i = z_j\} \ .$$

$i \setminus j$	F	M
F	0.03	−0.02
M	−0.50	0.15

Conclusion:
the gender effect is mainly,
that men seem not to like female friends.

Extended model specification

1. Endowment effect $g_i(\gamma, x, j)$

This represents the value of a tie
that is lost when the tie $i \rightarrow j$ is dissolved,
but that did not play a role when the tie was created.

This model component is used when certain effects
work differently for *creation* of ties ($0 \Rightarrow 1$)
than for *termination* of ties ($1 \Rightarrow 0$).

With this extension, actor i chooses to make the relational change that maximizes

$$f_i\big(\beta, x(i \rightsquigarrow j)\big) \; - \; x_{ij}\, g_i(\gamma, x, j) \; + \; U_i(t, x, j) \; .$$

The endowment function again can be a weighted sum

$$g_i(\gamma, x, j) = \sum_{h=1}^{H} \gamma_h \, r_{ijh}(x) \; .$$

Examples of components of endowment function:

1. $\gamma_1 \, x_{ji}$

   γ_1 extra benefits of a reciprocated tie.

Examples of components of endowment function:

1. $\gamma_1 \, x_{ji}$
   γ_1 extra benefits of a reciprocated tie.
2. $\gamma_2 \, w_{ij}$
   effect of dyadic covariate w_{ij}
   different for creating than for breaking a tie.
3. ... all other effects used also in the evaluation function.

Continuation example :

Reciprocity of a relation can have different effects
for creating than for breaking a relation.

Table next page:
total effect due to reciprocity conducive to *creating* a tie is 1.43;
total effect due to reciprocity
conducive to *breaking* a tie is $-1.43 - 1.21 = -2.64$.
Conclusion: reciprocity works stronger against terminating
than for creating reciprocated ties.

Add endowment effect of reciprocated tie

	Model 4	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	4.34	(0.71)
Rate $t_2 - t_3$	3.20	(0.51)
Out-degree	−1.60	(0.24)
Reciprocity	1.43	(0.39)
Transitive triplets	0.21	(0.11)
Indirect ties	−0.29	(0.07)
Sex (M) alter	0.53	(0.24)
Sex (M) ego	−0.48	(0.27)
Sex similarity	0.34	(0.26)
Program similarity	0.39	(0.14)
Smoking similarity	0.41	(0.19)
Endowment reciprocated tie	1.21	(0.46)

Extended model specification

2. Non-constant rate function $\lambda_i(\alpha, x)$.

This means that some actors change their ties
more quickly than others,
depending on covariates or network position.

Dependence on covariates:

$$\lambda_i(\alpha, x) = \rho_m \exp\Big(\sum_h \alpha_h v_{hi}\Big) .$$

ρ_m is a period-dependent base rate.

(Rate function must be positive; $\Rightarrow$ exponential function.)

Dependence on network position:
e.g., dependence on out-degrees:

$$\lambda_i(\alpha, x) = \exp(\alpha_1 \, x_{i+}) \ .$$

Also, in-degrees and $\sharp$ reciprocated ties of actor *i* may be used.

Now the parameter is $\theta = (\rho, \alpha, \beta, \gamma)$.

Further continuation example

Now add effect out-degrees on rate of change
and also endowment effect of reciprocity and sex similarity
(other similarity variables had no significant effects).

$$g_i(\gamma, x, j) = \gamma_1\, x_{ji} \;+\; \gamma_2\, \{1 - \mid w_i - w_j \mid\}$$

	Model 5	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	5.40	(0.82)
Rate $t_2 - t_3$	4.06	(0.63)
Out-degrees effect on rate	0.82	(0.52)
Out-degree	−1.44	(0.25)
Reciprocity	1.76	(0.43)
Transitive triplets	0.18	(0.12)
Indirect ties	−0.30	(0.07)
Sex (M) alter	0.51	(0.28)
Sex (M) ego	−0.37	(0.28)
Sex similarity	0.10	(0.38)
Program similarity	0.41	(0.13)
Smoking similarity	0.39	(0.19)
Endowment reciprocated tie	0.49	(0.47)
Endowment tie with other sex	−1.39	(0.66)

Conclusion:
endowment effect reciprocated tie becomes non-significant,
is taken over by endowment effect of tie to other sex.

Deleting (most) non-significant effects one by one
leads to following model.

Parsimonious model

	Model 6	
Effect	par.	(s.e.)
Rate $t_1 - t_2$	4.17	(0.69)
Rate $t_2 - t_3$	3.18	(0.53)
Out-degree	−1.52	(0.26)
Reciprocity	1.70	(0.36)
Transitive triplets	0.22	(0.11)
Indirect ties	−0.28	(0.07)
Sex (M) alter	0.54	(0.23)
Sex similarity	0.01	(0.35)
Program similarity	0.40	(0.13)
Smoking similarity	0.35	(0.19)
Endowment tie with other sex	−1.35	(0.61)

Final conclusion:
men more popular;
different-sex ties
are less stable.

2. Estimation

Suppose that at least 2 observations on $X(t)$ are available, for observation moments $t_1, \ t_2$.

How to estimate θ*?*

Condition on $X(t_1)$ *:*
the first observation is accepted as given,
contains in itself no observation about θ.

No assumption of a stationary network distribution.

Thus, simulations start with $X(t_1)$.

Method of moments

Choose a suitable statistic $Z = (Z_1, \ldots, Z_K)$,
i.e., K variables which can be calculated from the network;
the statistic Z must be *sensitive* to the parameter θ
in the sense that higher values of θ_k
lead to higher values of the expected value $\mathrm{E}_{\hat{\theta}}(Z_k)$;

determine value of $\theta = (\rho, \beta)$ for which
observed and expected values of suitable statistic are equal.

Questions:

- What is a suitable (K-dimensional) statistic?
  Corresponds to objective function.
- How to find this value of θ?
  By stochastic approximation (Robbins-Monro process)
  based on repeated simulations of the dynamic process,
  with parameter values
  getting closer and closer to the moment estimates.

Suitable statistics for method of moments

Assume first that $\lambda_i(x) = \rho = \theta_1$,
and 2 observation moments.

This parameter determines the expected "amount of change".

A sensitive statistic for $\theta_1 = \rho$ is

$$C = \sum_{\substack{i,j=1 \\ i \neq j}}^{g} \mid X_{ij}(t_2) - X_{ij}(t_1) \mid ,$$

the "observed total amount of change".

For the weights β_k in the evaluation function

$$f_i(\beta, x) = \sum_{k=1}^{L} \beta_k \, s_{ik}(x) \, ,$$

a higher value of β_k means that all actors strive more strongly after a high value of $s_{ik}(x)$, so $s_{ik}(x)$ will tend to be higher for all i, k.

This leads to the statistic

$$S_k = \sum_{i=1}^{n} s_{ik}(X(t_2)) \; .$$

This statistic will be sensitive to β_k :
a high β_k will to lead to high values of S_k.

Moment estimation will be based on the vector of statistics

$$Z = (C, S_1, ..., S_{K-1}) .$$

Denote by z the observed value for Z.
The moment estimate $\hat{\theta}$ is defined as the parameter value for which the expected value of the statistic is equal to the observed value:

$$\mathrm{E}_{\hat{\theta}}\{Z\} = z .$$

Robbins-Monro algorithm

The moment equation cannot be solved by analytical or the usual numerical procedures, because

$$\mathrm{E}_{\theta}\{Z\}$$

cannot be calculated explicitly.

However, the solution can be approximated by the Robbins-Monro (1951) method for stochastic approximation.

Iteration step:

$$\hat{\theta}_{N+1} = \hat{\theta}_N - a_N\, D^{-1}(z_N - z)\ , \tag{1}$$

where z_N is a simulation of Z with parameter $\hat{\theta}_N$,
D is a suitable matrix, and $a_N \rightarrow 0$.

Covariance matrix

The method of moments yields the covariance matrix

$$\text{cov}(\hat{\theta}) \approx D_\theta^{-1}\,\Sigma_\theta\,D_\theta'^{\,-1}$$

where

$$\begin{aligned}\Sigma_\theta &= \text{cov}\{Z \mid X(t_1) = x(t_1)\}\\ D_\theta &= \frac{\partial}{\partial\theta}\text{E}\{Z \mid X(t_1) = x(t_1)\}\,.\end{aligned}$$

(Note: Z is function of $X(t_1)$ and $X(t_2)$).

After the presumed convergence of the algorithm
for approximately solving the moment equation,
extra simulations are carried out

(*a*) to check that indeed $\mathrm{E}_{\hat{\theta}}\{Z\} \approx z$,

(*b*) to estimate Σ_θ,

(*c*) and to estimate D_θ
using a score function algorithm
(earlier algorithm used
difference quotients and common random numbers).

Modified estimation method:

conditional estimation .

Condition on the observed numbers of differences between successive observations,

$$c_m = \sum_{i,j} \mid x_{ij}(t_{m+1}) - x_{ij}(t_m) \mid \quad .$$

For continuing the simulations do not mind the values of the time variable t,
but continue between t_m and t_{m+1} until
the observed number of differences

$$\sum_{i,j} \mid X_{ij}(t) - x_{ij}(t_m) \mid$$

is equal to the observed c_m .
This is defined as time moment t_{m+1} .

This procedure is a bit more stable;
requires modified estimator of ρ_m .

Computer algorithm has 3 phases:

1. brief phase for preliminary estimation of $\partial \mathrm{E}_{\hat{\theta}}\{Z\}/\partial\theta$ for defining D;

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2. estimation phase with Robbins-Monro updates, where a_N remains constant in *subphases* and decreases between subphases;

Computer algorithm has 3 phases:

1. brief phase for preliminary estimation of $\partial \mathrm{E}_{\hat{\theta}}\{Z\}/\partial\theta$ for defining D;
2. estimation phase with Robbins-Monro updates, where a_N remains constant in *subphases* and decreases between subphases;
3. final phase where θ remains constant at estimated value; this phase is for checking that

$$\mathrm{E}_{\hat{\theta}}\{Z\} \approx z \; ,$$

and for estimating D_θ and Σ_θ to calculate standard errors.

Extension: more periods

The estimation method can be extended
to more than 2 repeated observations:
observations $x(t)$ for $t = t_1, ..., t_M$.

Parameters remain the same in periods between observations
except for the basic rate of change ρ
which now is given by ρ_m for $t_m \le t < t_{m+1}$.

For the simulations,
the simulated network $X(t)$ is reset to the observation $x(t_m)$
whenever the time parameter t passes the observation time t_m .

The statistics for the method of moments are defined as
sums of appropriate statistics calculated per period (t_m, t_{m+1}).

The procedure is implemented in the program

`S` imulation

`I` nvestigation for

`E` mpirical

`N` etwork

`A` nalysis

(new beta version 3.0) which can be downloaded from

`http://stat.gamma.rug.nl/snijders/siena.html`

(programmed by Tom Snijders, Christian Steglich, Michael Schweinberger, Mark Huisman).

A Windows shell is contained in the StOCNET package (new version 1.7) originally developed by Peter Boer (contributions by Bert Straatman, Evelien Zeggelink, Mark Huisman, Christian Steglich)

http://stat.gamma.rug.nl/stocnet/

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3. Networks as dependent and independent variables

Simultaneous endogenous dynamics of networks and behavior:
e.g.,

- individual humans & friendship relations:
  attitudes, behavior (lifestyle, health, etc.)
- individual humans & cooperation relations:
  work performance
- companies / organisations & alliances, cooperation:
  performance, organisational success.

Two-way influence between networks and behavior

Relational embeddedness is important
for well-being, opportunities, etc.

Actors are influenced in their behavior, attitudes, performance
by other actors to whom they are tied
e.g., network resources (social capital), social control.

(N. Friedkin, *A Structural Theory of Social Influence*, C.U.P., 1998).

In return, many types of tie
(friendship, cooperation, liking, etc.)
are influenced positively by
similarity on relevant attributes: *homophily*
(e.g., McPherson, Smith-Lovin, & Cook, *Ann. Rev. Soc.*, 2001.)

More generally, actors choose relation partners
on the basis of their behavior and other characteristics
(similarity, opportunities for future rewards, etc.).

Influence, network effects on behavior;
Selection, behavior effects on relations.

Terminology

relation = network = pattern of ties in group of actors;
behavior = any individual-bound changeable attribute
(including attitudes, performance, etc.).

Relations and behaviors are endogenous variables
that develop in a simultaneous dynamics.

Thus, there is a feedback relation in the dynamics
of relational networks and actor behavior / performance:
macro $\Rightarrow$ micro $\Rightarrow$ macro $\cdot\ \cdot\ \cdot$

The investigation of such social feedback processes is difficult:

- Both the *network* $\Rightarrow$ *behavior*
  and the *behavior* $\Rightarrow$ *network* effects
  lead 'network autocorrelation':
  "friends of smokers are smokers"
  "high-reputation firms don't collaborate
  with low-reputation firms".
  It is hard to ascertain the strengths
  of the causal relations in the two directions.
- For many phenomena
  quasi-continuous longitudinal observation is infeasible.
  Instead, it may be possible to observe
  networks and behaviors at a few discrete time points.

Data

One bounded set of actors
(e.g. school class, group of professionals, set of firms);

several discrete observation moments;

for each observation moment:

- network: who is tied to whom
- behavior of all actors

Aim: disentangle effects *networks* $\Rightarrow$ *behavior*
from effects *behavior* $\Rightarrow$ *networks*.

Notation:

Integrate the *influence* (network $\Rightarrow$ characteristics)
and *selection* (characteristics $\Rightarrow$ network) processes.

In addition to the network X, associated to each actor i
there is a vector $Z_i(t)$ of actor characteristics
indexed by $h = 1, \ldots, H$.
For the moment: ordered discrete
(simplest case: one dichotomous variable).

Actor-driven models

each actor “controls” not only his outgoing ties,
collected in the row vector $\big(X_{i1}(t), ..., X_{in}(t)\big)$,
but also his behavior $Z_i(t) = \big(\ Z_{i1}(t), ..., Z_{iH}(t)\ \big)$
(H is the number of dependent behavior variables).

Network change process and behavior change process
run simultaneously, and influence each other
being each other’s changing constraints.

At stochastic times
(*rate functions* λ^X for changes in network,
λ^Z_h for changes in behavior h),
the actors may change a tie or a behavior.

The actors try to attain a rewarding configuration
of the network and the behaviors
expressed in *evaluation functions* f^X and f^Z .
(Endowment functions may also be added.)

Again, conditional independence,
given the current network structure.

Functions λ^X , λ^Z , f^X , and f^Z depend on
K-dimensional statistical parameter $\theta \in \Theta \subset \mathrm{IR}^K$.

Micro-step for change in network:

At random moments occurring at a rate λ^X,
a random actor is designated
to make a change in one tie variable:
the *micro-step* (on $\Rightarrow$ off, or off $\Rightarrow$ on.)

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micro-step for change in behavior:

At random moments occurring at a rate λ^Z_h,
a random actor is designated to make a change in behavior h
(one component of Z_i, assumed to be ordinal):
the *micro-step* is a change to an adjacent category.

Again, many micro-steps can *accumulate* to big differences.

When actor i ‘may’ change an outgoing tie variable
to some other actor j,
he/she chooses the ’best’ j by maximizing
the evaluation function $f_i^X(\beta, X, z)$ of the situation obtained
after the coming network change
plus a random component representing unexplained influences;

and when this actor ‘may’ change behavior h,
he/she chooses the “best” change (up, down, nothing)
by maximizing the evaluation function $f_i^Z(\beta, x, Z)$ of the situation
obtained after the coming behavior change
plus a random component representing unexplained influences.

The new network is denoted by $x(i \leadsto j)$.
The attractiveness of the new situation
(evaluation function plus random term)
is expressed by the formula

$$f_i^X(\beta, x(i \leadsto j), z) \quad + \quad U_i^X(t, x, j)\,.$$

$$\Uparrow$$

random component

(Note that the network is also permitted to stay the same.)

Whenever actor i may make a change in variable h of Z,
he changes only one behavior, say z_{ih}, to the new value v.
The new vector is denoted by $z(i, h \rightsquigarrow v)$.
Actor i chooses the "best" h, v by maximizing
the evaluation function of the situation obtained
after the coming behavior change
plus a random component representing unexplained influences:

$$f_i^Z(\beta, x, z(i, h \rightsquigarrow v)) \quad + \quad U_i^Z(t, z, h, v)\,.$$

$$\Uparrow$$

random component

(behavior is permitted to stay the same.)

For the behaviors, the formula of the change probabilities is

$$p_{ihv}(\beta, z) = \frac{\exp(f(i,h,v))}{\sum_{k,u} \exp(f(i,k,u))}$$

where

$$f(i,h,v) = f_i^z(\beta, z(i, h \rightsquigarrow v)) \ .$$

Again, multinomial logit form.

Specification of the behavior model

Many different reasons why networks are important for behavior:

1. *social capital* :
   individuals may use resources of others;
2. *coordination* :
   individuals can achieve some goals
   only by concerted behavior;
3. *imitation* :
   individuals imitate others
   (basic drive; uncertainty reduction).

In this presentation, only imitation is considered,
but the other two reasons are also of eminent importance.

Basic effects for dynamics of behavior f_i^Z:

$$f_i^Z(\beta, x, z) = \sum_{k=1}^{L} \beta_k \, s_{ik}(x, z) \, ,$$

1. *tendency* ,
   $s_{i1}(x, z) = z_{ih}$
   A negative tendency parameter
   keeps and *sends* the behavior down;
   therefore, not only decreasing behavior values
   but also low average behavior values
   will yield a negative tendency parameter.

2. *behavior-related similarity*,
sum of behavior similarities
between i and his friends
$s_{i2}(x, z) = \sum_j x_{ij} \left(1 - | z_{ih} - z_{jh} |\right)$,
if Z_h assumes values between 0 and 1

2. *behavior-related similarity*,
   sum of behavior similarities
   between i and his friends
   $s_{i2}(x,z) = \sum_j x_{ij} \big(1 - |\, z_{ih} - z_{jh} \,|\big)$,
   if Z_h assumes values between 0 and 1
3. *average behavior alter* — an alternative to similarity:
   $s_{i3}(x,z) = z_{ih} \frac{1}{x_{i+}} \sum_j x_{ij} z_{jh}$

2. *behavior-related similarity*,
   sum of behavior similarities
   between i and his friends
   $s_{i2}(x, z) = \sum_j x_{ij} \left(1 - |\, z_{ih} - z_{jh} \,|\right)$,
   if Z_h assumes values between 0 and 1
3. *average behavior alter* — an alternative to similarity:
   $s_{i3}(x, z) = z_{ih} \frac{1}{x_{i+}} \sum_j x_{ij} z_{jh}$
4. *popularity-related tendency*, (in-degree)
   $s_{i4}(x, z) = z_{ih} \, x_{+i}$

2. *behavior-related similarity*,
   sum of behavior similarities
   between i and his friends
   $s_{i2}(x,z) = \sum_j x_{ij} \left(1 - |\, z_{ih} - z_{jh} \,|\right)$,
   if Z_h assumes values between 0 and 1
3. *average behavior alter* — an alternative to similarity:
   $s_{i3}(x,z) = z_{ih} \frac{1}{x_{i+}} \sum_j x_{ij} z_{jh}$
4. *popularity-related tendency*, (in-degree)
   $s_{i4}(x,z) = z_{ih}\, x_{+i}$
5. *activity-related tendency*, (out-degree)
   $s_{i5}(x,z) = z_{ih}\, x_{i+}$

Effects 2 and 3 are alternatives for each other:
they express the same theoretical idea of influence
in mathematically different ways.
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6. *dependence on other behaviors* $(h \neq \ell)$,
   $s_{i6}(x,z) = z_{ih}\, z_{i\ell}$

For both the network and the behavior dynamics,
extensions are possible depending on the network position.

Now focus on the *similarity effect* in evaluation function :

sum of absolute behavior differences between *i* and his friends
$s_{i2}(x, z) = \sum_j x_{ij} \big(1 - |\, z_{ih} - z_{jh} \,|\big)$.

This is fundamental both
to network selection based on behavior,
and to behavior change based on network position.

A positive coefficient for this effect means that the actors prefer friends with similar Z_h values (*network autocorrelation*).

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A positive coefficient for this effect means that the actors prefer friends with similar Z_h values (*network autocorrelation*).

Actors can attempt to attain this by changing their own Z_h value to the average value of their friends (*network influence*, *contagion*),

or by becoming friends with those with similar Z_h values (*selection on similarity*).

Consider two behaviors Z_1 and Z_2 , for both of which
the actors prefer positive network autocorrelation.

It is very well conceivable that actors attempt to reach this
by network changes for Z_1 and behavior changes for Z_2 .

E.g., for a friendship relation:
let Z_1 be religion and Z_2 musical taste.

E.g., for cooperation between firms:
let Z_1 be location and Z_2 administrative organization.

Since the functions f^X and f^Z
express the trade-off between rewards and costs,
even when the rewards are the same,
the costs of change may well be different.

Statistical estimation: networks & behavior

Procedures for estimating parameters in this model are similar to estimation procedures for network-only dynamics: Methods of Moments & Stochastic Approximation, conditioning on the first observation $X(t_1), Z(t_1)$.

The two different effects,
networks $\Rightarrow$ behavior and behavior $\Rightarrow$ networks,
both lead to network autocorrelation of behavior;

but they can be (in principle)
distinguished empirically by the time order: respectively association between ties at t_m and behavior at t_{m+1};
and association between behavior at t_m and ties at t_{m+1}.

Statistics for use in method of moments:

for estimating parameters in network dynamics:

$$\sum_{m=1}^{M-1} \sum_{i=1}^{n} s_{ik}(X(t_{m+1}), Z(t_m)) \ ,$$

and for the behavior dynamics:

$$\sum_{m=1}^{M-1} \sum_{i=1}^{n} s_{ik}(X(t_m), Z(t_{m+1})) \ .$$

The data requirements for these models are strong:
few missing data; enough change on the behavioral variable.

Currently, work still is going on about good ways
for estimating parameters in these models.

Maximum likelihood estimation procedures
(currently even more time-consuming; under construction...)
are preferable for small data sets.

Example :

Study of smoking initiation and friendship
in a Scottish secondary school
(following up on earlier work by P. West, M. Pearson & others).
One school year group from a Scottish secondary school
starting at age 12-13 years, was monitored over 3 years,
129 pupils present at all 3 observations,
with sociometric & behavior questionnaires
at three moments, at appr. 1 year intervals.

What does this data set tell us about the mutual effects
between friendship and smoking?

First, results for a model
for dynamics in networks and in smoking behavior
under the assumption that both are unrelated.

Smoking measured in three categories:
1 = no, 2 = occasionally, 3 = regularly.

There is more network change than behavior change;
$\Rightarrow$ more power for network than behavior dynamics.

In this group, girls smoke more than boys.

Parameter estimates for network dynamics: assumed independent of smoking.

	Effect	Estimate	Standard error
Rate function			
λ_0^X	Rate parameter t_1–t_2	11.88	1.20
λ_1^X	Rate parameter t_2–t_3	9.48	0.96
evaluation function			
β_1^X	Out-degree	–2.49	0.11
β_2^X	Reciprocity	2.06	0.08
β_3^X	Number of distances 2	–0.81	0.01
β_4^X	Transitive triplets	0.17	0.01
β_5^X	Gender (*F*) alter	–0.23	0.17
β_6^X	Gender (*F*) ego	0.16	0.07
β_7^X	Gender similarity	0.79	0.09
β_8^X	Classmates	0.04	0.03

Parameter estimates for smoking dynamics:
assumed independent of friendship.

	Effect	Estimate	Standard error
Rate function			
λ_0^Z	Rate parameter t_1–t_2	0.78	0.26
λ_1^Z	Rate parameter t_2–t_3	0.79	0.24
evaluation function			
β_1^Z	Tendency	–0.49	0.19
β_2^Z	Gender (*F*)	0.85	0.37
β_3^Z	Parents' smoking	–0.44	0.46
β_4^Z	Siblings' smoking	1.05	0.41

Now a similar analysis,
but with a model in which there is a mutual effect
between smoking and friendship formation.

Parameter estimates network dynamics dependent on smoking.

	Effect	Estimate	Standard error
Rate function			
λ_0^X	Rate parameter t_1–t_2	11.84	1.34
λ_1^X	Rate parameter t_2–t_3	9.64	1.04
evaluation function			
β_1^X	Out-degree	–2.28	0.29
β_2^X	Reciprocity	2.07	0.16
β_3^X	Number of distances 2	–0.86	0.07
β_4^X	Transitive triplets	0.15	0.08
β_5^X	Gender (*F*) alter	–0.17	0.09
β_6^X	Gender (*F*) ego	0.16	0.07
β_7^X	Gender similarity	0.77	0.11
β_8^X	Classmates	0.05	0.03
β_9^X	Smoking similarity	0.19	0.08
β_{10}^X	Smoking alter	–0.14	0.01
β_{11}^X	Smoking ego	0.05	0.16

Parameter estimates for smoking dynamics: dependent on friendship.

	Effect	Estimate	Standard error
Rate function			
λ_0^Z	Rate parameter t_1–t_2	0.90	0.31
λ_1^Z	Rate parameter t_2–t_3	0.87	0.25
evaluation function			
β_1^Z	Tendency	0.01	0.29
β_2^Z	Gender (F)	0.60	0.41
β_3^Z	Parents' smoking	–0.52	0.50
β_4^Z	Siblings' smoking	1.04	0.50
β_5^Z	Similarity to friends	0.61	0.44

Conclusions :

Evidence for effect of smoking on friendship development;

no evidence for effect of friendship on smoking initiation;

smokers less popular as friends.

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Evidence for effect of smoking on friendship development;

no evidence for effect of friendship on smoking initiation;

smokers less popular as friends.

Taking the mutual effect of smoking and friendship into account (even though effect friendship $\Rightarrow$ smoking is not significant) reduces strongly the estimated effect of gender:

in the first analysis
there seemed some evidence for a gender effect on smoking, but this might also be explained as an effect of friendship (friendship is rather gender-homogeneous).

A multivariate analysis with two dependent behavior variables,
smoking and alcohol consumption,
gives a similar picture for smoking,
but there is evidence for social influence on drinking behavior;
alcohol-based selection is stronger
than smoking-based selection;
pupils drinking more are *more* popular;
drinking has a positive effect on smoking, but not vice versa.

Discussion issues

- Software: SIENA
- Work on applications
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- the *fit* of these models
  and the *robustness* of the conclusions;
- richer modeling of network effects is important:
  e.g., of network positions of individual actors
  (cf. Pearson & West, *Connections*, 2003.)

Other work (current or near future)

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8. Relations with more than 2 ordered values.
9. Multivariate relations.
10. Fit diagnostics.

Additional references

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